

INTRODUCTION *to*

POST-TONAL
THEORY

4th
EDITION

Joseph N. Straus

Introduction to Post-Tonal Theory

4th
EDITION

Introduction to Post-Tonal Theory

4th
EDITION

JOSEPH N. STRAUS The Graduate Center, City University of New York



W. W. NORTON & COMPANY
NEW YORK • LONDON
WWW.WWNORTON.COM

W. W. Norton & Company has been independent since its founding in 1923, when William Warder Norton and Mary D. Herter Norton first published lectures delivered at the People's Institute, the adult education division of New York City's Cooper Union. The firm soon expanded its program beyond the Institute, publishing books by celebrated academics from America and abroad. By midcentury, the two major pillars of Norton's publishing program—trade books and college texts—were firmly established. In the 1950s, the Norton family transferred control of the company to its employees, and today—with a staff of four hundred and a comparable number of trade, college, and professional titles published each year—W. W. Norton & Company stands as the largest and oldest publishing house owned wholly by its employees.

Copyright © 2016, 2005, 2000, 1990 by Joseph N. Straus

This book was previously published by: Pearson Education, Inc.

All rights reserved

Printed in the United States of America

Editor: Justin Hoffman

Project Editors: Debra Nichols, Rachel Mayer

Editorial Assistant: Grant Phelps

Manuscript Editor: Jodi Beder

Managing Editor, College: Marian Johnson

Managing Editor, College Digital Media: Kim Yi

Associate Director of Production, College: Benjamin Reynolds

Media Editor: Steve Hoge

Media Editorial Assistant: Stephanie Eads

Marketing Manager, Music: Trevor Penland

Design Director: Jillian Burr

Designer: Wendy Lai

Permissions Manager: Megan Jackson

Composition: codeMantra

Manufacturing: R. R. Donnelley & Sons—Kendallville IN

Permission to use copyrighted material begins on p. 385.

Names: Straus, Joseph Nathan, author.

Title: Introduction to post-tonal theory / Joseph N. Straus.

Description: Fourth edition. | New York : W.W. Norton & Company, 2016. | Includes bibliographical references and index.

Identifiers: LCCN 2015047742 | **ISBN 978-0-393-93883-8 (hardcover)**

Subjects: LCSH: Music theory. | Atonality. | Twelve-tone system. | Musical analysis.

Classification: LCC MT40.S96 2016 | DDC 781.2/67—dc23 LC record available at <http://lccn.loc.gov/2015047742>

W. W. Norton & Company, Inc., 500 Fifth Avenue, New York, NY 10110-0017

wwnorton.com

W. W. Norton & Company Ltd., Castle House, 75/76 Wells Street, London W1T 3QT

CONTENTS IN BRIEF

Contents	vi
Preface	xiii
1 Basic Concepts of Pitch and Interval	1
2 Pitch-Class Sets	43
3 Some Additional Properties and Relationships	95
4 Motive, Voice Leading, and Harmony	159
5 Centricity and Referential Pitch Collections	228
6 Basic Concepts of Twelve-Tone Music	294
<i>List of Set Classes</i>	378
Answers to Selected Exercises	382
Credits	385
Index of Concepts	389
Index of Composers and Works.....	395

CONTENTS

1 Basic Concepts of Pitch and Interval 1

- 1.1 Octave Equivalence 1
- 1.2 Enharmonic Equivalence 3
- 1.3 Pitch and Pitch Class 4
- 1.4 Integer Notation 5
- 1.5 Arithmetic *modulo 12* (mod 12) 6
- 1.6 Intervals (Calculated in Semitones) 7
- 1.7 Pitch Intervals (Ordered and Unordered) 9
- 1.8 Ordered Pitch-Class Intervals 9
- 1.9 Unordered Pitch-Class Intervals 11
- 1.10 Interval Class 12
- 1.11 Interval-Class Content 13
- 1.12 Interval-Class Vector 16
- 1.13 Spacing and Register 17

EXERCISES 18

MODEL ANALYSES 22

- 1.1 Anton Webern, “Wie bin ich froh!” from *Three Songs*, op. 25 22
- 1.2 Arnold Schoenberg, “Nacht” (Night), from *Pierrot Lunaire*, op. 21 28

GUIDED ANALYSES 31

- 1.1 Anton Webern, *Symphony*, op. 21, second movement, Theme 32
- 1.2 Arnold Schoenberg, *Piano Concerto*, op. 42, first movement 33
- 1.3 Igor Stravinsky, “Musick to heare,” from *Three Shakespeare Songs* 33
- 1.4 Ruth Crawford Seeger, *Diaphonic Suite No. 1* (for oboe or flute), first movement 35
- 1.5 Edgard Varèse, *Octandre*, first movement 35
- 1.6 Milton Babbitt, “The Widow’s Lament in Springtime” 36
- 1.7 Luciano Berio, *Sequenza I* (for solo flute) 37
- 1.8 Kaija Saariaho, *NoaNoa* (for flute and live electronics) 38
- 1.9 Elliott Carter, *Riconoscenza per Goffredo Petrassi* (for solo violin) 39
- 1.10 Ursula Mamlok, *Variations for Solo Flute*, Theme 41

BIBLIOGRAPHY AND FURTHER READING 42

- 2.1 Pitch-Class Sets 43
- 2.2 Normal Form 45
 - 2.2.1 Putting a set into normal form 45
 - 2.2.2 Using the pitch-class clockface 46
- 2.3 Transposition (T_n) 46
 - 2.3.1 Line (or series) of pitches 46
 - 2.3.2 Line (or series) of pitch classes 47
 - 2.3.3 Set of pitch classes 48
 - 2.3.4 Levels of transposition 49
 - 2.3.5 Transposing a pitch-class set 49
 - 2.3.6 Recognizing sets related by transposition 50
 - 2.3.7 Nodes, arrows, and networks 51
 - 2.3.8 Inverse 52
- 2.4 Inversion (I_n) 53
 - 2.4.1 Index number (sum) 53
 - 2.4.2 Line (or series) of pitches 55
 - 2.4.3 Line (or series) of pitch classes 55
 - 2.4.4 Set of pitch classes 56
- 2.5 Inversion (I_5) 58
- 2.6 Set Class 62
- 2.7 Prime Form 66
- 2.8 *List of Set Classes* 68
- 2.9 Segmentation and Analysis 69

EXERCISES 71**MODEL ANALYSES 75**

- 2.1 Arnold Schoenberg, *Book of the Hanging Gardens*, op. 15, no. 11 75
- 2.2 Béla Bartók, *String Quartet No. 4*, first movement 81

GUIDED ANALYSES 86

- 2.1 Ruth Crawford Seeger, *Piano Prelude No. 9* 86
- 2.2 Anton Webern, *Concerto for Nine Instruments*, op. 24, second movement 87
- 2.3 Igor Stravinsky, *Agon*, “Pas de deux” 88
- 2.4 Anton Webern, *Five Movements for String Quartet*, op. 5, no. 2 90
- 2.5 Milton Babbitt, *Semi-Simple Variations*, Theme 90
- 2.6 Igor Stravinsky, *Three Pieces for String Quartet*, no. 2 91

BIBLIOGRAPHY AND FURTHER READING 93

- 3.1 Common Tones Under Transposition (T_n) 96
 - 3.1.1 Interval-class content 96
 - 3.1.2 Some special set classes (major scale and whole-tone scale) 98
- 3.2 Transpositional Symmetry 100
 - 3.2.1 Interval-class vector 101
 - 3.2.2 Some special set classes 101
 - 3.2.3 The pitch-class clockface 102
 - 3.2.4 Degrees of transpositional symmetry 103
- 3.3 Common Tones Under Inversion (I_n) 103
 - 3.3.1 Calculating common tones under inversion 104
 - 3.3.2 Common tones as a source of musical continuity 105
- 3.4 Inversional Symmetry 107
 - 3.4.1 Intervallic palindrome (mirror image) 107
 - 3.4.2 Pitch symmetry and pitch-class symmetry 108
 - 3.4.3 Degrees of inversional symmetry 110
- 3.5 Symmetry and Set Class 112
- 3.6 Z-relation 112
 - 3.6.1 Z-correspondents 113
 - 3.6.2 The two “all-interval” tetrachords 113
- 3.7 Complement Relation 115
 - 3.7.1 Interval content 115
 - 3.7.2 Literal and abstract complements 117
 - 3.7.3 *List of Set Classes* 119
 - 3.7.4 Hexachords 119
- 3.8 Inclusion Relation (Subsets and Supersets) 121
 - 3.8.1 Subsets of the same type 121
 - 3.8.2 Inclusion lattice 122
 - 3.8.3 Projecting subsets in pitch space 122
 - 3.8.4 Literal and abstract subsets and supersets 123
- 3.9 Transpositional Combination (TC) 124
- 3.10 Contour Relations 126
 - 3.10.1 Contour segment (CSEG) 127
 - 3.10.2 CSEG-class 127
 - 3.10.3 Contours of dynamics and durations 129

EXERCISES 132

MODEL ANALYSES 137

- 3.1 Anton Webern, *Movements for String Quartet*, op. 5, no. 4 137
- 3.2 Alban Berg, “Schlafend trägt man mich,” from *Four Songs*, op. 2, no. 2 143

GUIDED ANALYSES 148

- 3.1 Igor Stravinsky, *Agon*, “Bransle Gay” 148
- 3.2 Arnold Schoenberg, *Six Little Piano Pieces*, op. 19, no. 2 150
- 3.3 George Crumb, *Vox Balaenae* (Voice of the Whale), “Vocalise (. . . for the Beginning of Time)” 152
- 3.4 György Ligeti, *Étude 11: En Suspens* 154
- 3.5 Anton Webern, *Five Pieces for Orchestra*, op. 10, no. 4 156

BIBLIOGRAPHY AND FURTHER READING 157

4

Motive, Voice Leading, and Harmony 159

- 4.1 Composing-Out 159
- 4.2 Interval Cycles 163
 - 4.2.1 Cyclic linear motion 165
 - 4.2.2 Cyclic harmonies 166
 - 4.2.3 Maximal evenness 169
 - 4.2.4 Combination cycles 171
- 4.3 Voice Leading 174
 - 4.3.1 Transposition and inversion 175
 - 4.3.2 Fuzzy transposition and inversion (with offset) 177
- 4.4 Set-Class Space 179
 - 4.4.1 Voice-leading space for trichords 179
 - 4.4.2 Voice-leading space for tetrachords 181
 - 4.4.3 Harmonic quality 182
- 4.5 Contextual Inversion 183
 - 4.5.1 Chain and space for (014) 183
 - 4.5.2 Chain and space for B–G–A–B^b 185
- 4.6 Triadic Post-Tonality 188
 - 4.6.1 Triadic transformation 188
 - 4.6.2 Other progressions of triads 196

EXERCISES 199

MODEL ANALYSES 202

- 4.1 Stravinsky, *The Rite of Spring*, Part 1, Introduction 202
- 4.2 Anton Webern, *Little Pieces for Cello and Piano*, op. 11, no. 1 207

GUIDED ANALYSES 214

- 4.1 Thomas Adès, *The Tempest*, Act III, Scene 5 214
- 4.2 Karlheinz Stockhausen, *Klavierstück III* 217
- 4.3 Béla Bartók, String Quartet No. 3, *Prima parte* 218
- 4.4 Elliott Carter, *Scrivo in Vento* 220
- 4.5 Witold Lutosławski, *Funeral Music*, first movement 221
- 4.6 Arnold Schoenberg, *Five Piano Pieces*, op. 23, no. 3 223
- 4.7 Alfred Schnittke, *Hymnus II* (for cello and double bass) 224

BIBLIOGRAPHY AND FURTHER READING 225

5

Centricity and Referential Pitch Collections 228

- 5.1 Tonality and Centricity 228
- 5.2 Inversional Axis 232
 - 5.2.1 Inversional symmetry in pitch space 232
 - 5.2.2 Inversional wedges 234
 - 5.2.3 Inversional symmetry in pitch-class space (axis of symmetry) 236
 - 5.2.4 Motion from axis to axis 241
- 5.3 Diatonic Collection 244
- 5.4 Octatonic Collection 249
- 5.5 Whole-Tone Collection 252
- 5.6 Hexatonic Collection 257
- 5.7 Collectional and Centric Interaction 260

EXERCISES 263

MODEL ANALYSES 265

- 5.1 Igor Stravinsky, *Petrushka*, Scene 1 265
- 5.2 Béla Bartók, *Piano Sonata*, first movement, first theme and second theme 273

GUIDED ANALYSES 277

- 5.1 Igor Stravinsky, *Oedipus Rex* 277
- 5.2 Igor Stravinsky, *Orpheus* 278
- 5.3 Igor Stravinsky, “In a foolish dream,” from *The Rake’s Progress* 279
- 5.4 Béla Bartók, *Mikrokosmos*, no. 109, “From the Island of Bali” 281
- 5.5 Béla Bartók, *Mikrokosmos*, no. 101, “Diminished Fifth” 283
- 5.6 Ellen Taaffe Zwilich, String Quartet No. 2, second movement 284
- 5.7 Peter Maxwell Davies, *Eight Songs for a Mad King*, no. 3, “The Lady-in-Waiting: Miss Musgrave’s Fancy” 285
- 5.8 Claude Debussy, “Voiles” 286
- 5.9 Anton Webern, *Six Bagatelles*, op. 9, no. 5 287

- 5.10 Olivier Messiaen, “Amen du Désir,” from *Visions de l’Amen*
(for two pianos) 289
- 5.11 Joan Tower, “Vast Antique Cubes,” from *No Longer Very Clear*
(for piano solo) 290

BIBLIOGRAPHY AND FURTHER READING 292

6

Basic Concepts of Twelve-Tone Music 294

- 6.1 Twelve-Tone Series 294
 - 6.1.1 Set and series 294
 - 6.1.2 Role of the series 295
- 6.2 Basic Operations 295
 - 6.2.1 Content and order 295
 - 6.2.2 Prime ordering 296
 - 6.2.3 Transposition 297
 - 6.2.4 Retrograde 297
 - 6.2.5 Inversion 298
 - 6.2.6 Retrograde-inversion 299
 - 6.2.7 Series class (row class) 299
 - 6.2.8 12×12 matrix 301
 - 6.2.9 “Twelve-count” 303
 - 6.2.10 Composing with a series 306
- 6.3 Segmental Subsets 307
 - 6.3.1 Direct presentation 308
 - 6.3.2 Indirect presentation 309
- 6.4 Invariants 311
 - 6.4.1 Invariant dyads 312
 - 6.4.2 Invariant trichords 314
 - 6.4.3 Invariant dyads between series forms 315
- 6.5 Varieties of Twelve-Tone Music 318
 - 6.5.1 Webern and derivation 318
 - 6.5.2 Schoenberg and hexachordal combinatoriality 322
 - 6.5.3 Stravinsky and rotational arrays 328
 - 6.5.4 Crawford Seeger and multilevel rotation 332
 - 6.5.5 Babbitt and trichordal arrays 334

EXERCISES 338

MODEL ANALYSES 342

- 6.1 Anton Webern, String Quartet, op. 28, first movement, Variation 4 342
- 6.2 Arnold Schoenberg, *Piano Piece*, op. 33a, first theme 347

GUIDED ANALYSES 354

- 6.1 Arnold Schoenberg, *Suite for Piano*, op. 25, Gavotte 354

- 6.2 Igor Stravinsky, *In Memoriam Dylan Thomas*, Prelude
("Dirge-Canons") 356
- 6.3 Anton Webern, *Quartet*, op. 22, first movement 357
- 6.4 Luigi Dallapiccola, *Goethe Lieder*, no. 2, "Die Sonne kommt!" 358
- 6.5 Luigi Dallapiccola, *Quaderno Musicale di Annalibera*, "Contrapunctus
Secundus (canon contrario motu)" 360
- 6.6 Igor Stravinsky, *Epitaphium* 362
- 6.7 Ursula Mamlok, *Panta Rhei*, third movement 363
- 6.8 Charles Wuorinen, *Twelve Pieces*, no. 3 365
- 6.9 Anton Webern, *Concerto for Nine Instruments*, op. 24, third
movement 366
- 6.10 Arnold Schoenberg, *String Quartet No. 4*, first movement 367
- 6.11 Igor Stravinsky, *Requiem Canticles*, "Exaudi" 369
- 6.12 Ruth Crawford Seeger, *Three Songs*, "Prayers of Steel" 372
- 6.13 Milton Babbitt, *Du*, Song No. 1 ("Wiedersehen") 374

BIBLIOGRAPHY AND FURTHER READING 376

PREFACE

A broad consensus has emerged among music theorists regarding the basic musical elements of post-tonal music—pitch, interval, motive, harmony—and this book reports that consensus to a general audience of musicians and students of music. Like books on scales, triads, and simple harmonic progressions in tonal music, this book introduces basic theoretical concepts for the post-tonal music of the twentieth and twenty-first centuries.

Beyond basic concepts, this fourth edition also contains information on many of the most recent developments in post-tonal theory, including expanded or new coverage of the following topics:

- transformational networks and graphs
- composing-out
- transformational voice leading
- voice-leading space
- harmonic quality
- triadic post-tonality
- inversional symmetry and inversional axes
- interval cycles
- contextual inversion and inversional chains

As a result, this book is not only a primer of basic concepts but also an introduction to the current state of post-tonal theory, with its rich array of theoretical concepts and analytical tools.

Like previous editions of *Introduction to Post-Tonal Theory*, each chapter of the text features a clear and concise exposition of theoretical topics. New pedagogical aids enhance the fourth edition: each chapter begins with an outline of its content, and In Brief boxes summarize each section. Each chapter concludes with exercises; selected answers are provided at the back of the book so that students can check to see if they're on the right track.

This fourth edition features new analysis pedagogy. After the theoretical exercises in each chapter, you'll find two Model Analyses that apply the theoretical principles elucidated in the chapter. These are followed by Guided Analyses, where students are presented with musical passages of modest length and prompted with a series of analytical questions. These Guided Analyses are suitable both for written assignments (of a variety of lengths) and classroom discussion. They offer students a chance to apply the theoretical concepts they've seen in the chapter and Model Analyses. They also offer a forum for the discussion of questions of form, rhythm, and expression.

In the Guided Analyses, questions are designed to stimulate analytical thought, not to suggest definitive, correct answers. The music discussed in this book is inherently challenging and precludes simple answers. There are many possible responses to these questions and many possible interpretations of the relationships in this music.

In addition to these substantial pedagogical changes, I have also made some modest changes in the presentation of theoretical material. In discussing inversion, I have retired the traditional compound operation $T_n I_n$, in favor of the simpler I_n and (I_n^2) models. Following the suggestion of Aleck Brinkman, who prepared the *List of Set Classes*, I have changed and simplified the procedure for putting sets in normal form. Finally, following the emerging practice in the professional literature, I make relatively infrequent use of Forte-names for set classes.

Although the “classical” prewar repertoire of music by Schoenberg, Stravinsky, Bartók, Webern, and Berg still comprises the musical core, illustrations of theoretical concepts and Guided Analyses now include music by a wide variety of composers, including Adams, Adès, Babbitt, Berio, Boulez, Britten, Cage, Carter, Cowell, Crawford, Crumb, Dallapiccola, Davies, Debussy, Feldman, Glass, Gubaidulina, Ives, Ligeti, Lutosławski, Mamlök, Messiaen, Musgrave, Reich, Ruggles, Saariaho, Schnittke, Sessions, Shostakovich, Stockhausen, Tower, Varèse, Wolpe, Wuorinen, and Zwilich.

In preparing the fourth edition, I received invaluable advice from Gretchen Foley, Dave Headlam, Julian Hook, Steven Rings, and Matthew Santa. I am deeply grateful to these experienced scholars and sorry I could not take even more of their superb suggestions. The manuscript was class tested by three more veterans—Cynthia Folio, Jonathan Pieslak, and David Schober—and I am grateful to them and to their students at Temple University and the CUNY Graduate Center. I received editorial and notational help from six brilliant doctoral students at the CUNY Graduate Center: Megan Lavengood, Christina Lee, Tim Mastic, Simon Prosser, Noel Torres-Rivera, and Abby Zhang. Lori Wacker prepared an earlier version of the Answer Key. At Norton, Maribeth Payne welcomed me to a wonderful new publishing home, and Justin Hoffman guided the project to completion with his customary grace and incisiveness. Rachel Mayer project edited, Jodi Beder copy-edited, Debra Nichols project edited and proofread, Benjamin Reynolds was the production manager, and Grant Phelps was the editorial assistant. For me, they have been an editorial dream team. Closer to home, in matters both tangible and intangible, Sally Goldfarb has offered continuing guidance and support beyond my ability to describe or repay.

JOSEPH N. STRAUS
Graduate Center
City University of New York

Introduction to Post-Tonal Theory

4th
EDITION

Basic Concepts of Pitch and Interval

1

In this chapter, you will learn some standard ways of thinking about pitch and interval in post-tonal music, with the intervals counted in semitones.

- 1.1 Octave Equivalence
- 1.2 Enharmonic Equivalence
- 1.3 Pitch and Pitch Class
- 1.4 Integer Notation
- 1.5 Arithmetic *modulo 12* (mod 12)
- 1.6 Intervals (Calculated in Semitones)
- 1.7 Pitch Intervals (Ordered and Unordered)
- 1.8 Ordered Pitch-Class Intervals
- 1.9 Unordered Pitch-Class Intervals
- 1.10 Interval Class
- 1.11 Interval-Class Content
- 1.12 Interval-Class Vector
- 1.13 Spacing and Register

1.1 OCTAVE EQUIVALENCE

Pitches separated by one or more perfect octaves are usually understood as *equivalent*. Our musical notation reflects that equivalence by giving the same name to octave-related pitches. The name A, for example, is given not only to some particular pitch (for example, the A that lies a minor third below middle C), but also to all the other pitches one or more octaves above or below it. Octave-related pitches are called by the same name because they sound so much alike and because Western music usually treats them as functionally equivalent.

Things that are equivalent are not necessarily identical, however. **Example 1-1** shows two versions of a melody that are different in many ways, particularly in their rhythm and range. The range of the second version is so wide that the first violin cannot reach all of the notes; the cello has to step in to help. At the same time, however, it is easy to recognize that they are basically the same melody, because they are *octave equivalent*.

EXAMPLE 1-1 Two octave-equivalent melodies (Schoenberg, *String Quartet No. 4*, first movement).

The image shows two staves of music. The top staff is labeled 'Violin 1' and contains a melodic line in 4/4 time. The bottom staff is labeled 'Cello' and contains a lower melodic line. Dotted lines connect notes between the two staves, illustrating their octave equivalence. The notes in the violin staff are G4, B4, and G5. The notes in the cello staff are G2, B2, and G3. The key signature has one sharp (F#) and the time signature is 4/4.

In **Example 1-2**, compare the first three notes of the melody with the sustained notes in measures 4–5. There are many differences between the two collections of notes (register, articulation, rhythm, etc.), but there is also a basic equivalence: they both contain a B, a G#, and a G.

EXAMPLE 1-2 Two octave-equivalent musical ideas (Schoenberg, *Three Piano Pieces*, op. 11, no. 1).

The image shows a piano score in 3/4 time. The top staff is labeled 'Piano' and contains a melodic line. The bottom staff contains sustained notes. The melodic line starts with a circled group of notes: B4, G#4, and G5. The sustained notes in the bottom staff are B2, G#2, and G3. The key signature has one sharp (F#) and the time signature is 3/4. The word 'Mäßige' is written above the first staff, and a piano dynamic marking 'p' is present.

We find the same situation in **Example 1-3**: the first three notes of the viola melody—G, B, and C#—return as the cadential chord at the end of the phrase. The melody and the chord are *octave equivalent*.

EXAMPLE 1-3 Two octave-equivalent musical ideas (Webern, *Movements for String Quartet*, op. 5, no. 2).

Sehr langsam (♩ = ca. 56)

IN BRIEF

Pitches that are one (or more than one) octave apart may be considered *equivalent*.

1.2 ENHARMONIC EQUIVALENCE

In common-practice tonal music, B \flat is not the same as A \sharp . Even on an equal-tempered instrument like the piano, the tonal system gives B \flat and A \sharp different functions and meanings. In G major, for example, B \flat is $\hat{b}3$ whereas A \sharp is $\hat{\sharp}2$, and these different scale degrees have very different musical roles. These distinctions are largely abandoned in post-tonal music, however, where notes that are *enharmonically equivalent* (like B \flat and A \sharp) are also functionally equivalent.

For example, the passage in **Example 1-4** involves three repetitions: the A returns an octave higher, the B returns two octaves lower, and the A \flat returns three octaves higher as a G \sharp . All three pairs of notes are octave equivalent; in addition, A \flat and G \sharp are *enharmonically equivalent*.

EXAMPLE 1-4 Enharmonic equivalence (Stockhausen, *Klavierstück III*).

There may be isolated moments where a composer notates a pitch in what seems like a functional way (sharps for ascending motion and flats for descending, for example). For the most part, however, the notation of post-tonal music is functionally arbitrary, determined by convenience and legibility.

IN BRIEF

Notes that are *enharmonically equivalent* (like B \flat and A \sharp) may be considered equivalent.

1.3 PITCH AND PITCH CLASS

By invoking octave and enharmonic equivalence, we can distinguish between a *pitch* (a note with a certain frequency) and a *pitch class* (a group of pitches with the same or enharmonic name). Pitch-class A, for example, contains all the pitches named A, and any pitch named A is a member or representative of pitch-class A. When we say that the lowest note on the cello is a C, we are referring to a specific pitch. We can notate that pitch on the second ledger line beneath the bass staff. And we can refer to it using the numerical designation of middle C as C₄—the lowest note on the cello is thus C₂. When we say that the tonic of Beethoven's Fifth Symphony is C, however, we are referring not to some particular *pitch* C, but to *pitch-class* C. Pitch-class C is an abstraction and cannot be adequately notated on musical staves. Sometimes, for convenience, we will represent a pitch class using musical notation. In reality, however, a pitch class is not a single thing; it is a class of things: namely, pitches one or more octaves apart.

In **Example 1-5**, each of the three instruments plays a series of notes. We hear many different pitches as the instrumental lines leap about. The tuba, for example, plays five different *pitches*, most of which are repeated. But taking the passage as a whole, we hear only three *pitch classes*: F \sharp , G, and A \flat .

EXAMPLE 1-5 Many pitches, but only three pitch classes: F \sharp , G, and A \flat (Feldman, *Durations III*, No. 3).

Note: The violin is playing harmonics that produce pitches two octaves higher than the filled-in noteheads.

The musical score consists of three staves: Violin, Tuba, and Piano. The Violin staff is marked 'Slow' and contains a sequence of notes with various accidentals (flats and sharps) and a '15ma' marking. The Tuba staff shows notes with accidentals. The Piano staff shows notes with accidentals and a '15ma' marking.

A *pitch class* is a collection of pitches related by octave and enharmonic equivalence.

1.4 INTEGER NOTATION

There are only twelve pitch classes. All the B[#]s, C[♮]s, and D[♭]s are members of a single pitch class, as are all the C[♯]s and D[♭]s, all the C[×]s, D[♯]s, and E[♭]s, and so on. We will often use *integers* from 0 through 11 to refer to the twelve pitch classes. **Example 1-6** shows the twelve pitch classes and some of the contents of each, following a “fixed *do*” notation: the pitch class containing the C[♮] is arbitrarily assigned the integer 0, and the rest follow from there.

EXAMPLE 1-6 Integer notation of pitch class.

Integer Name	Pitch-Class Content
0	B [#] , C, D [♭]
1	C [#] , D [♭]
2	C [×] , D, E [♭]
3	D [#] , E [♭]
4	D [×] , E, F [♭]
5	E [#] , F, G [♭]
6	F [#] , G [♭]
7	F [×] , G, A [♭]
8	G [#] , A [♭]
9	G [×] , A, B [♭]
10	A [#] , B [♭]
11	A [×] , B, C [♭]

When referring to pitch classes, we will use either traditional letter names or *pitch-class integers*, whichever seems clearest and easiest in a particular context. In **Example 1-7**, pitch-class integers are assigned to the notated pitches (with octave and enharmonic equivalence assumed throughout).

EXAMPLE 1-7 Integer notation of pitch class (Babbitt, *Composition for Four Instruments*).

Clarinet

$\text{♩} = 120$

mp 11 3 0 *f* *ff* *f* 8^b 4 *mp*

4 10 9 7 2 5

7 1 11 10 3 0 2 8 4
mf *p* *ff* *ppp* *mf* *p* *ff*

Integers are traditional in music theory (as figured-bass numbers, for example) and useful for representing certain musical relationships. We will never do things to the integers that don't have musical significance; rather, we will use numbers and arithmetic to help us think about aspects of the music we study. The music itself is not “mathematical” any more than our lives are “mathematical” because we count our ages in integers.

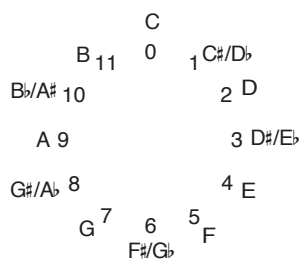
IN BRIEF

Each of the twelve pitch classes is identified by an *integer*, 0–11.

1.5 ARITHMETIC *MODULO* 12 (MOD 12)

Every pitch belongs to one of the twelve pitch classes. Going up an octave (adding twelve semitones) or going down an octave (subtracting twelve semitones) will produce another member of the same pitch class. For example, if we start on the E^b above middle C (a member of pitch-class 3) and go up twelve semitones, we end up back on pitch-class 3. In other words, in the world of twelve pitch classes, $3 + 12 = 15 = 3$.

Any number larger than 11 or smaller than 0 is equivalent to some integer from 0 to 11. To figure out which one, just add or subtract 12 (or any multiple of 12). Twelve is called the *modulus*, and we will frequently use arithmetic *modulo 12*, for which *mod 12* is an abbreviation. In a *mod 12 system*, $-12 = 0 = 12 = 24$, and so on. Similarly, $-13, -1, 23$, and 35 are all equivalent to 11 (and to each other) because they are related to 11 (and to each other) by adding or subtracting 12. It is easiest to understand these (and other) mod 12 relationships by envisioning a circular *clockface*, like the one in **Example 1-8**.

EXAMPLE 1-8 The *pitch-class clockface*.

We locate pitches in an extended *pitch space*, ranging in equal-tempered semitones from the lowest to the highest audible tone. The traditional grand staff is a good illustration of pitch space: it provides distinct positions for all of the pitches, including the 88 pitches represented by the keys of the piano keyboard. In contrast, we locate pitch classes in a modular *pitch-class space*, as in **Example 1-8**, which circles back on itself and contains only the twelve pitch classes. You can imagine that the linear pitch space of the staff has been wrapped around onto the circular pitch-class space of the clockface. It's like the hours of the day or the days of the week. As our lives unfold in time, each hour and each day are uniquely located in linear time, never to be repeated. But we can be sure that, if it's eleven o'clock now, it will be eleven o'clock in twelve hours (that's a mod 12 system), and that if it's Friday today, it will be Friday again in seven days (that's a mod 7 system). Just as our lives unfold simultaneously in linear and modular time, music unfolds simultaneously in pitch and pitch-class space.

IN BRIEF

In *modular pitch-class space* (represented by the *pitch-class clockface*), going up or down by twelve semitones leads to another member of the same pitch class.

1.6 INTERVALS (CALCULATED IN SEMITONES)

In tonal music, the interval between two pitches is named with reference to steps in a diatonic scale (e.g., major *third*, diminished *fifth*). Post-tonal music, however, doesn't necessarily refer to diatonic scales, so the traditional interval names can be cumbersome or even misleading. Rather, intervals in post-tonal music are named by the number of semitones they contain. Just as A# and Bb are part of the same pitch class, the major third and diminished fourth are treated as the same interval, because both contain four semitones.

Example 1-9 shows a series of seven harmonic intervals played in rhythmic unison. The first six intervals are spelled as major thirds while the seventh is spelled as a diminished fourth, but in this musical context it is clear that all seven intervals are to be understood as enharmonically equivalent: all are 4s, or compound (i.e., octave-equivalent) 4s.

A *pitch interval* (*pi*) is the distance between two pitches, measured by the number of semitones between them. A pitch interval is created when we move from pitch to pitch in *pitch space*. It can be as large as the range of our hearing or as small as a semitone. Sometimes we will be concerned with the direction of the interval, whether ascending or descending. In that case, the number will be preceded by either a plus sign (to indicate an ascending interval) or a minus sign (to indicate a descending interval). Intervals with a plus or minus sign are called *directed* or *ordered pitch intervals* (*opi*). At other times, we will be concerned only with the absolute space between two pitches. For such *unordered pitch intervals* (*upi*), we will just provide the number of semitones between the pitches. For example, when we say that C₄ ascends four semitones to E₄, we are talking about an ordered pitch interval (*opi* = +4); when we say that there are four semitones between C₄ and E₄, we are talking about an unordered pitch interval (*upi* = 4).

Whether we consider an interval ordered or unordered depends on our particular analytical interests. **Example 1-11** identifies both ordered and unordered pitch intervals. The *ordered pitch intervals* focus attention on the contour of the line, its balance of rising and falling motion. The *unordered pitch intervals* ignore contour and concentrate on the spaces between the pitches.

EXAMPLE 1-11 Ordered and unordered pitch intervals (Schoenberg, *String Quartet No. 3*, first movement).

ordered pitch intervals:	-1	+3	-5	-6	+15	-6	-5	+8	-4
--------------------------	----	----	----	----	-----	----	----	----	----

unordered pitch intervals:	1	3	5	6	15	6	5	8	4
----------------------------	---	---	---	---	----	---	---	---	---

IN BRIEF

A *pitch interval* is the interval between two pitches, and may be understood either as *ordered* (i.e., ascending or descending) or *unordered* (the space between the notes without respect to direction).

A *pitch-class interval* (*pci*) is the distance between two pitch classes. A pitch-class interval is created when we move from pitch class to pitch class in modular pitch-class space. It can never be larger than eleven semitones, because no two pitch classes can be more than eleven semitones apart. As with pitch intervals, we will sometimes be concerned with *ordered pitch-class intervals* (*opci*) and sometimes with *unordered pitch-class intervals* (*upci*).