INTRODUCTION *to* POST-TONAL THEORY

4th EDITION

Joseph N. Straus

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JOSEPH N. STRAUS The Graduate Center, City University of New York

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CONTENTS IN BRIEF

CONTENTS

2 Pitch-Class Sets 43

3 Some Additional Properties and Relationships 95

PREFACE

A broad consensus has emerged among music theorists regarding the basic musical elements of post-tonal music—pitch, interval, motive, harmony—and this book reports that consensus to a general audience of musicians and students of music. Like books on scales, triads, and simple harmonic progressions in tonal music, this book introduces basic theoretical concepts for the post-tonal music of the twentieth and twenty-first centuries.

Beyond basic concepts, this fourth edition also contains information on many of the most recent developments in post-tonal theory, including expanded or new coverage of the following topics:

- transformational networks and graphs
- composing-out
- transformational voice leading
- voice-leading space
- harmonic quality
- triadic post-tonality
- inversional symmetry and inversional axes
- interval cycles
- contextual inversion and inversional chains

As a result, this book is not only a primer of basic concepts but also an introduction to the current state of post-tonal theory, with its rich array of theoretical concepts and analytical tools.

Like previous editions of *Introduction to Post-Tonal Theory*, each chapter of the text features a clear and concise exposition of theoretical topics. New pedagogical aids enhance the fourth edition: each chapter begins with an outline of its content, and In Brief boxes summarize each section. Each chapter concludes with exercises; selected answers are provided at the back of the book so that students can check to see if they're on the right track.

This fourth edition features new analysis pedagogy. After the theoretical exercises in each chapter, you'll find two Model Analyses that apply the theoretical principles elucidated in the chapter. These are followed by Guided Analyses, where students are presented with musical passages of modest length and prompted with a series of analytical questions. These Guided Analyses are suitable both for written assignments (of a variety of lengths) and classroom discussion. They offer students a chance to apply the theoretical concepts they've seen in the chapter and Model Analyses. They also offer a forum for the discussion of questions of form, rhythm, and expression.

In the Guided Analyses, questions are designed to stimulate analytical thought, not to suggest definitive, correct answers. The music discussed in this book is inherently challenging and precludes simple answers. There are many possible responses to these questions and many possible interpretations of the relationships in this music.

In addition to these substantial pedagogical changes, I have also made some modest changes in the presentation of theoretical material. In discussing inversion, I have retired the traditional compound operation T_nI, in favor of the simpler I_n and (I_y^x) models. Following the suggestion of Aleck Brinkman, who prepared the *List of Set Classes*, I have changed and simplified the procedure for putting sets in normal form. Finally, following the emerging practice in the professional literature, I make relatively infrequent use of Forte-names for set classes.

Although the "classical" prewar repertoire of music by Schoenberg, Stravinsky, Bartók, Webern, and Berg still comprises the musical core, illustrations of theoretical concepts and Guided Analyses now include music by a wide variety of composers, including Adams, Adès, Babbitt, Berio, Boulez, Britten, Cage, Carter, Cowell, Crawford, Crumb, Dallapiccola, Davies, Debussy, Feldman, Glass, Gubaidulina, Ives, Ligeti, Lutosławski, Mamlok, Messiaen, Musgrave, Reich, Ruggles, Saariaho, Schnittke, Sessions, Shostakovich, Stockhausen, Tower, Varèse, Wolpe, Wuorinen, and Zwilich.

In preparing the fourth edition, I received invaluable advice from Gretchen Foley, Dave Headlam, Julian Hook, Steven Rings, and Matthew Santa. I am deeply grateful to these experienced scholars and sorry I could not take even more of their superb suggestions. The manuscript was class tested by three more veterans—Cynthia Folio, Jonathan Pieslak, and David Schober—and I am grateful to them and to their students at Temple University and the CUNY Graduate Center. I received editorial and notational help from six brilliant doctoral students at the CUNY Graduate Center: Megan Lavengood, Christina Lee, Tim Mastic, Simon Prosser, Noel Torres-Rivera, and Abby Zhang. Lori Wacker prepared an earlier version of the Answer Key. At Norton, Maribeth Payne welcomed me to a wonderful new publishing home, and Justin Hoffman guided the project to completion with his customary grace and incisiveness. Rachel Mayer project edited, Jodi Beder copyedited, Debra Nichols project edited and proofread, Benjamin Reynolds was the production manager, and Grant Phelps was the editorial assistant. For me, they have been an editorial dream team. Closer to home, in matters both tangible and intangible, Sally Goldfarb has offered continuing guidance and support beyond my ability to describe or repay.

JOSEPH N. STRAUS
Graduate Center
City University of New York

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Basic Concepts of Pitch and Interval

1

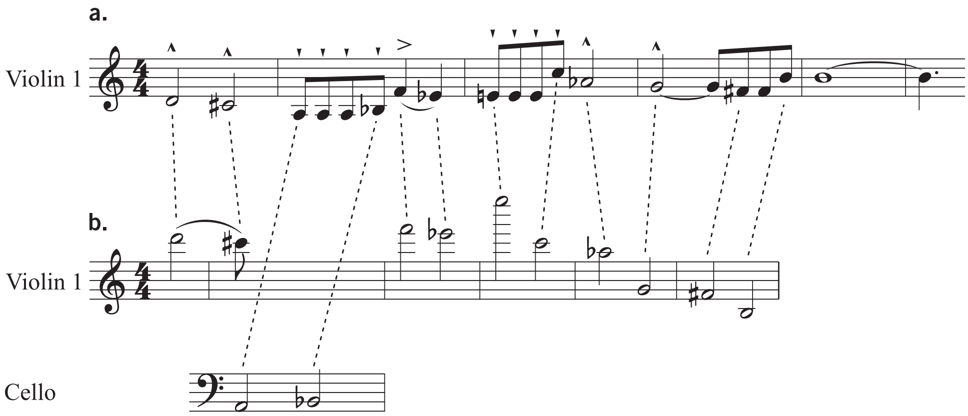
In this chapter, you will learn some standard ways of thinking about pitch and interval in post-tonal music, with the intervals counted in semitones.

1.1 OCTAVE EQUIVALENCE

Pitches separated by one or more perfect octaves are usually understood as ***equivalent***. Our musical notation reflects that equivalence by giving the same name to octave-related pitches. The name A, for example, is given not only to some particular pitch (for example, the A that lies a minor third below middle C), but also to all the other pitches one or more octaves above or below it. Octave-related pitches are called by the same name because they sound so much alike and because Western music usually treats them as functionally equivalent.

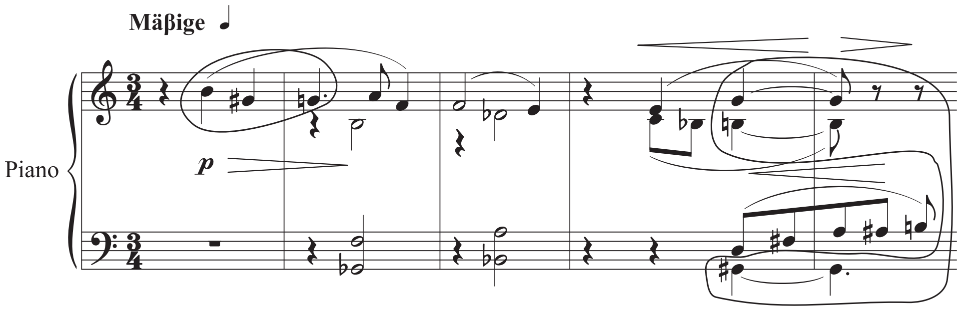
Things that are equivalent are not necessarily identical, however. **Example 1-1** shows two versions of a melody that are different in many ways, particularly in their rhythm and range. The range of the second version is so wide that the first violin cannot reach all of the notes; the cello has to step in to help. At the same time, however, it is easy to recognize that they are basically the same melody, because they are ***octave equivalent***.

EXAMPLE 1-1 **Two octave-equivalent melodies (Schoenberg, String Quartet No. 4, first movement).**



In **Example 1-2**, compare the first three notes of the melody with the sustained notes in measures 4–5. There are many differences between the two collections of notes (register, articulation, rhythm, etc.), but there is also a basic equivalence: they both contain a B, a G♯, and a G.

EXAMPLE 1-2 **Two octave-equivalent musical ideas (Schoenberg, *Three Piano Pieces,* op. 11, no. 1).**



We find the same situation in **Example 1-3**: the first three notes of the viola melody—G, B, and C♯—return as the cadential chord at the end of the phrase. The melody and the chord are *octave equivalent*.

EXAMPLE 1-3 Two octave-equivalent musical ideas (Webern, *Movements for String Quartet,* op. 5, no. 2).

IN BRIEF

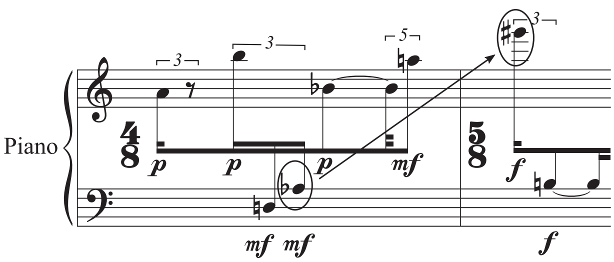
Pitches that are one (or more than one) octave apart may be considered *equivalent*.

1.2 ENHARMONIC EQUIVALENCE

In common-practice tonal music, B♭ is not the same as A♯. Even on an equal-tempered instrument like the piano, the tonal system gives B♭ and A♯ different functions and meanings. In G major, for example, B♭ is ♭$\hat{3}$ whereas A♯ is ♯$\hat{2}$, and these different scale degrees have very different musical roles. These distinctions are largely abandoned in post-tonal music, however, where notes that are ***enharmonically equivalent*** (like B♭ and A♯) are also functionally equivalent.

For example, the passage in **Example 1-4** involves three repetitions: the A returns an octave higher, the B returns two octaves lower, and the A♭ returns three octaves higher as a G♯. All three pairs of notes are octave equivalent; in addition, A♭ and G♯ are *enharmonically equivalent*.

EXAMPLE 1-4 Enharmonic equivalence (Stockhausen, *Klavierstuck III*).



There may be isolated moments where a composer notates a pitch in what seems like a functional way (sharps for ascending motion and flats for descending, for example). For the most part, however, the notation of post-tonal music is functionally arbitrary, determined by convenience and legibility.

IN BRIEF

Notes that are *enharmonically equivalent* (like B♭ and A♯) may be considered equivalent.

1.3 PITCH AND PITCH CLASS

By invoking octave and enharmonic equivalence, we can distinguish between a *pitch* (a note with a certain frequency) and a ***pitch class*** (a group of pitches with the same or enharmonic name). Pitch-class A, for example, contains all the pitches named A, and any pitch named A is a member or representative of pitch-class A. When we say that the lowest note on the cello is a C, we are referring to a specific pitch. We can notate that pitch on the second ledger line beneath the bass staff. And we can refer to it using the numerical designation of middle C as C4—the lowest note on the cello is thus C2. When we say that the tonic of Beethoven's Fifth Symphony is C, however, we are referring not to some particular *pitch* C, but to *pitch-class* C. Pitch-class C is an abstraction and cannot be adequately notated on musical staves. Sometimes, for convenience, we will represent a pitch class using musical notation. In reality, however, a pitch class is not a single thing; it is a class of things: namely, pitches one or more octaves apart.

In **Example 1-5**, each of the three instruments plays a series of notes. We hear many different pitches as the instrumental lines leap about. The tuba, for example, plays five different *pitches*, most of which are repeated. But taking the passage as a whole, we hear only three *pitch classes*: F♯, G, and A♭.

EXAMPLE 1-5 **Many pitches, but only three pitch classes: F♯, G, and A♭ (Feldman, *Durations III*, No. 3). Note: The violin is playing harmonics that produce pitches two octaves higher than the filled-in noteheads.**

IN BRIEF

A *pitch class* is a collection of pitches related by octave and enharmonic equivalence.

1.4 INTEGER NOTATION

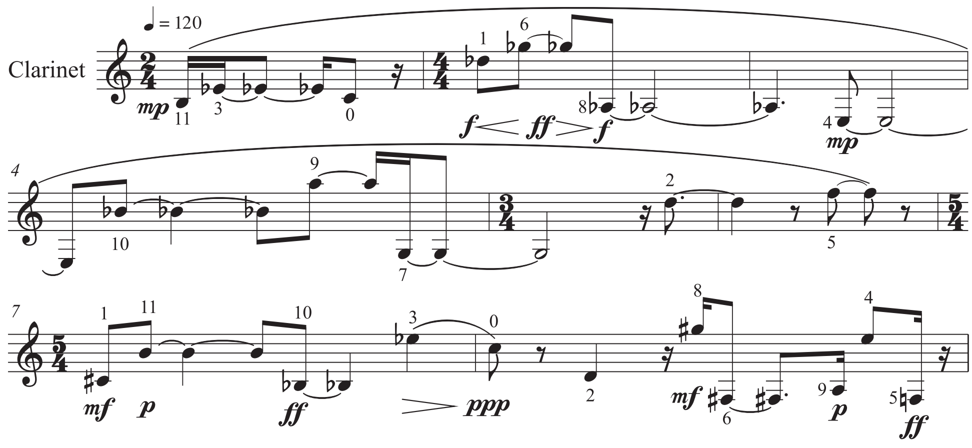
There are only twelve pitch classes. All the B♯s, C♮s, and D♭♭s are members of a single pitch class, as are all the C♯s and D♭s, all the C𝄪s, D♮s, and E♭♭s, and so on. We will often use *integers* from 0 through 11 to refer to the twelve pitch classes. **Example 1-6** shows the twelve pitch classes and some of the contents of each, following a "fixed *do*" notation: the pitch class containing the Cs is arbitrarily assigned the integer 0, and the rest follow from there.

EXAMPLE 1-6 **Integer notation of pitch class.**

Integer Name	Pitch-Class Content
0	B♯, C, D♭♭
1	C♯, D♭
2	C𝄪, D, E♭♭
3	D♯, E♭
4	D𝄪, E, F♭
5	E♯, F, G♭♭
6	F♯, G♭
7	F𝄪, G, A♭♭
8	G♯, A♭
9	G𝄪, A, B♭♭
10	A♯, B♭
11	A𝄪, B, C♭

When referring to pitch classes, we will use either traditional letter names or ***pitch-class integers***, whichever seems clearest and easiest in a particular context. In **Example 1-7**, pitch-class integers are assigned to the notated pitches (with octave and enharmonic equivalence assumed throughout).

EXAMPLE 1-7 **Integer notation of pitch class (Babbitt, *Composition for Four Instruments*).**



Integers are traditional in music theory (as figured-bass numbers, for example) and useful for representing certain musical relationships. We will never do things to the integers that don't have musical significance; rather, we will use numbers and arithmetic to help us think about aspects of the music we study. The music itself is not "mathematical" any more than our lives are "mathematical" because we count our ages in integers.

IN BRIEF

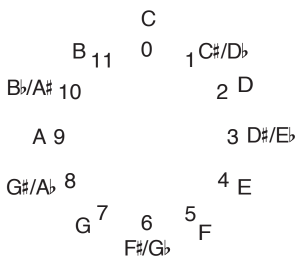
Each of the twelve pitch classes is identified by an *integer*, 0–11.

1.5 ARITHMETIC *MODULO 12* (MOD 12)

Every pitch belongs to one of the twelve pitch classes. Going up an octave (adding twelve semitones) or going down an octave (subtracting twelve semitones) will produce another member of the same pitch class. For example, if we start on the E♭ above middle C (a member of pitch-class 3) and go up twelve semitones, we end up back on pitch-class 3. In other words, in the world of twelve pitch classes, 3 + 12 = 15 = 3.

Any number larger than 11 or smaller than 0 is equivalent to some integer from 0 to 11. To figure out which one, just add or subtract 12 (or any multiple of 12). Twelve is called the *modulus,* and we will frequently use arithmetic ***modulo 12***, for which ***mod 12*** is an abbreviation. In a *mod 12 system*, –12 = 0 = 12 = 24, and so on. Similarly, –13, –1, 23, and 35 are all equivalent to 11 (and to each other) because they are related to 11 (and to each other) by adding or subtracting 12. It is easiest to understand these (and other) mod 12 relationships by envisioning a circular *clockface*, like the one in **Example 1-8**.

EXAMPLE 1-8 ***The pitch-class clockface.***



We locate pitches in an extended ***pitch space,*** ranging in equal-tempered semitones from the lowest to the highest audible tone. The traditional grand staff is a good illustration of pitch space: it provides distinct positions for all of the pitches, including the 88 pitches represented by the keys of the piano keyboard. In contrast, we locate pitch classes in a modular *pitch-class space,* as in **Example 1-8**, which circles back on itself and contains only the twelve pitch classes. You can imagine that the linear pitch space of the staff has been wrapped around onto the circular pitch-class space of the clockface. It's like the hours of the day or the days of the week. As our lives unfold in time, each hour and each day are uniquely located in linear time, never to be repeated. But we can be sure that, if it's eleven o'clock now, it will be eleven o'clock in twelve hours (that's a mod 12 system), and that if it's Friday today, it will be Friday again in seven days (that's a mod 7 system). Just as our lives unfold simultaneously in linear and modular time, music unfolds simultaneously in pitch and pitch-class space.

IN BRIEF

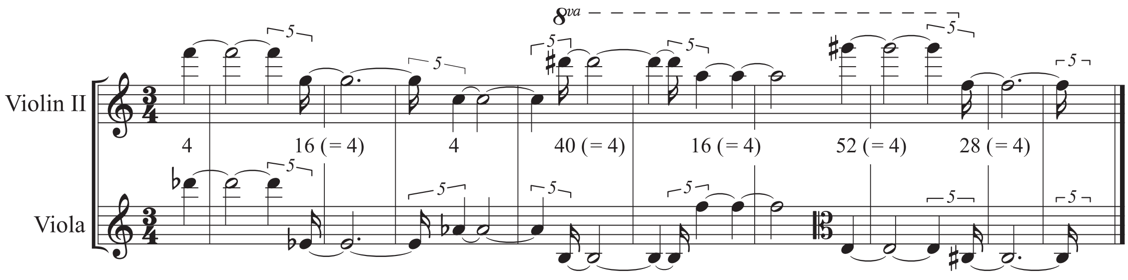
In *modular pitch-class space* (represented by the *pitch-class clockface*), going up or down by twelve semitones leads to another member of the same pitch class.

1.6 INTERVALS (CALCULATED IN SEMITONES)

In tonal music, the interval between two pitches is named with reference to steps in a diatonic scale (e.g., major *third*, diminished *fifth*). Post-tonal music, however, doesn't necessarily refer to diatonic scales, so the traditional interval names can be cumbersome or even misleading. Rather, intervals in post-tonal music are named by the number of semitones they contain. Just as A♯ and B♭ are part of the same pitch class, the major third and diminished fourth are treated as the same interval, because both contain four semitones.

Example 1-9 shows a series of seven harmonic intervals played in rhythmic unison. The first six intervals are spelled as major thirds while the seventh is spelled as a diminished fourth, but in this musical context it is clear that all seven intervals are to be understood as enharmonically equivalent: all are 4s, or compound (i.e., octave-equivalent) 4s.

EXAMPLE 1-9 **Enharmonic and octave-equivalent intervals (Carter, String Quartet No. 3).**



Example 1-10 gives some traditional interval names and the number of semitones they contain.

EXAMPLE 1-10 **Intervals counted in semitones.**

Traditional Name	No. of Semitones
unison	0
minor 2nd	1
major 2nd, diminished 3rd	2
minor 3rd, augmented 2nd	3
major 3rd, diminished 4th	4
augmented 3rd, perfect 4th	5
augmented 4th, diminished 5th	6
perfect 5th, diminished 6th	7
augmented 5th, minor 6th	8
major 6th, diminished 7th	9
augmented 6th, minor 7th	10
major 7th	11
octave	12
minor 9th	13
major 9th	14
minor 10th	15
major 10th	16

IN BRIEF

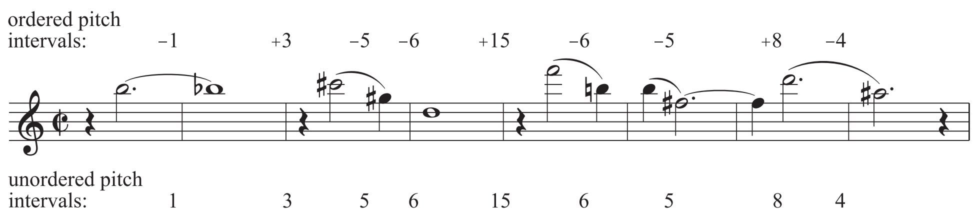
Intervals are calculated in *semitones*, not in diatonic steps or with traditional designations of quality (like major and minor).

1.7 PITCH INTERVALS (ORDERED AND UNORDERED)

A ***pitch interval*** (***pi***) is the distance between two pitches, measured by the number of semitones between them. A pitch interval is created when we move from pitch to pitch in *pitch space*. It can be as large as the range of our hearing or as small as a semitone. Sometimes we will be concerned with the direction of the interval, whether ascending or descending. In that case, the number will be preceded by either a plus sign (to indicate an ascending interval) or a minus sign (to indicate a descending interval). Intervals with a plus or minus sign are called ***directed*** or ***ordered pitch intervals (opi)***. At other times, we will be concerned only with the absolute space between two pitches. For such ***unordered pitch intervals (upi)***, we will just provide the number of semitones between the pitches. For example, when we say that C4 ascends four semitones to E4, we are talking about an ordered pitch interval (opi = +4); when we say that there are four semitones between C4 and E4, we are talking about an unordered pitch interval (upi = 4).

Whether we consider an interval ordered or unordered depends on our particular analytical interests. **Example 1-11** identifies both ordered and unordered pitch intervals. The *ordered pitch intervals* focus attention on the contour of the line, its balance of rising and falling motion. The *unordered pitch intervals* ignore contour and concentrate on the spaces between the pitches.

EXAMPLE 1-11 **Ordered and unordered pitch intervals (Schoenberg, String Quartet No. 3, first movement).**



IN BRIEF

A *pitch interval* is the interval between two pitches, and may be understood either as *ordered* (i.e., ascending or descending) or *unordered* (the space between the notes without respect to direction).

1.8 ORDERED PITCH-CLASS INTERVALS

A ***pitch-class interval*** (***pci***) is the distance between two pitch classes. A pitch-class interval is created when we move from pitch class to pitch class in modular pitch-class space. It can never be larger than eleven semitones, because no two pitch classes can be more than eleven semitones apart. As with pitch intervals, we will sometimes be concerned with ***ordered pitch-class intervals*** (***opci***) and sometimes with ***unordered pitch-class intervals*** (***upci***).